

GGSIPIU mathematics 2007

1. If $p \wedge \sim r \rightarrow \sim p \wedge q$ is false, then the truth values of p, q and r are respectively

- a T, F and F b F, F and T
c F, T and T d T, F and T

2. If α, β and γ are the roots of equation $x^3 - 8x + 8 = 0$, then $\sum \alpha^2$ and $\sum \frac{1}{\alpha\beta}$ are respectively

- a 0 and -16 b 16 and 8
c -16 and 0 d 16 and 0

3. The GCD of 1080 and 675 is

- a 145 b 135
c 225 d 125

4. If a, b and c $\in \mathbb{N}$, then which one of the following is not true ?

- a $a \mid b$ and $a \mid c \Rightarrow a \mid 3b + 2c$
b $a \mid b$ and $a \mid c \Rightarrow a \mid c$
c $a \mid b + c \Rightarrow a \mid ba$ and $a \mid c$
d $a \mid b$ and $a \mid c \Rightarrow a \mid b + c$

5. $x = 4 + \cos \theta$ and $y = 1 + \sin \theta$ are the parametric equations of

- a $\frac{(x-3)^2}{9} + \frac{(y-4)^2}{16} = 1$
b $\frac{(x+4)^2}{16} + \frac{(y+3)^2}{9} = 1$
c $\frac{(x-4)^2}{16} - \frac{(y-3)^2}{9} = 1$
d $\frac{(x-4)^2}{16} + \frac{(y-3)^2}{9} = 1$

6. If the distance between the foci and the distance between the directrices of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are in the ratio 3:2, then a:b is

- a $\sqrt{2} : 1$

b $\sqrt{3} : \sqrt{2}$

c 1:2

(c) 2:2

7. The ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ have in common

a centre only

b centre, foci and directrices

c centre, foci and vertices

d centre and vertices only

8. If $\sec \theta = m$ and $\tan \theta = n$, then $\frac{1}{m} [m+n + \frac{1}{m+n}]$ is

a 2 b 2m

(c) 2n (d) m

9. The value of $\frac{\sin 85^\circ - \sin 35^\circ}{\cos 65^\circ}$ is

a 2 b -1

c 1 d 0

10. If the length of the tangent from any point on the circle $x^2 + y^2 = 5r^2$ to the circle $x^2 + y^2 = r^2$ is 16 unit, then the area between the two circles in sq unit is

a 32π b 4π

b 8π d 256π

11. The equation of the common tangent of the two touching circles $y^2 + x^2 - 6x - 12y + 37 = 0$ and $x^2 + y^2 - 6y + 7 = 0$ is

a $x+y-5=0$ b $x-y+5=0$

c $x-y-5=0$ d $x+y+5=0$

12. The equation of the parabolas with vertex at $(-1, 1)$ and focus $(2, 1)$ is

a $y^2 - 2y - 12x - 11 = 0$

b $x^2 + 2x - 12y + 13 = 0$

$$c \ y^2 - 2y + 12x + 11 = 0$$

$$d \ y^2 - 2y - 12x + 13 = 0$$

13. The equation of the line which is tangent to both the circle $x^2 + y^2 = 5$ and the parabola $y^2 = 40x$ is

a $2x - y + 5 = 0$

b $2x - y + 5 = 0$

c $2x - y - 5 = 0$

(d) $2x + 5 = 0$

14. If $2A + 3B = \begin{bmatrix} 2 & -1 & 4 \\ 3 & 2 & 4 \end{bmatrix}$ and $A + 2B = \begin{bmatrix} 5 & 0 & 3 \\ 1 & 6 & 2 \end{bmatrix}$, then B is

a $\begin{bmatrix} 8 & -1 & 2 \\ -1 & 10 & -1 \end{bmatrix}$ (l) $\begin{bmatrix} 8 & 1 & 2 \\ -1 & 10 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} 8 & 1 & 2 \\ -1 & 10 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 8 & 1 & 2 \\ 1 & 10 & 1 \end{bmatrix}$

15. If $A = \begin{bmatrix} 1 & -3 \\ 2 & k \end{bmatrix}$ and $A^2 - 4A + 10I = A$, then k is equal to

a 0 b -4

c) 4 and not 1 d) 1 or 4

16. The value of $\begin{vmatrix} x+y & y+z & z+x \\ x & y & z \\ x-y & y-z & z-x \end{vmatrix}$ is equal to

a $2x+y+z$ b $2x+y+z$

c $x+y+z$ d 0

17. On the set Q of all rational numbers the operation * which is both associative and commutative is given by $a*b$, is

a $a+b+ab$ b a^2+b^2

c $ab+1$ d $2a+3b$

18. From an aeroplane flying, vertically above a horizontal road, the angles of depression of two consecutive stones on the same side of aeroplane are observed to be 30° and 60° respectively. The height at which the aeroplane is flying in km is

a $\frac{4}{\sqrt{3}}$

b $\frac{\sqrt{3}}{2}$

c $\frac{2}{\sqrt{3}}$ d 2

19. If the angles of a triangle are in the ratio 3:4:5, then the sides are in the ratio

a 2: $\sqrt{6}$: $\sqrt{3+1}$ b $\sqrt{2}$: $\sqrt{6}$: $\sqrt{3+1}$

c 2: $\sqrt{3}$: $\sqrt{3+1}$ d 3:4:5

20. If $\cos^{-1} x = \alpha$, $0 < x < 1$ and $\sin^{-1} 2x \sqrt{1-x^2} + \sec^{-1} \frac{1}{2x^2-1} = \frac{2\pi}{3}$, then $\tan^{-1} 2x$ equals

a $\frac{\pi}{6}$ b $\frac{\pi}{4}$

c $\frac{\pi}{3}$ d $\frac{\pi}{2}$

21. If $a > b > 0$, then the value of $\tan^{-1} \left(\frac{a}{b} \right) + \tan^{-1} \left(\frac{a+b}{a-b} \right)$ depends on

a both a and b b b and not a

c a and not b d neither a nor b

22. If $A = \{a, b, c\}$, $B = \{b, c, d\}$ and $C = \{a, d, c\}$, then $A \cap B \cap C$ is equal to

a $\{a, c, a, d\}$

b $\{a, b, c, d\}$

c $\{c, a, d, a\}$

d $\{a, c, a, d, b, d\}$

23. The function $f: X \rightarrow Y$ defined by $f(x) = \sin x$ is one one but not onto, if X and Y are respectively equal to

a \mathbb{R} and \mathbb{R}

b $[0, \pi]$ and $[0, 1]$

c $[0, \frac{\pi}{2}]$ and $[-1, 1]$

d $[\frac{-\pi}{2}, \frac{\pi}{2}]$ and $[-1, 1]$

24. If $\log_4 2 + \log_4 4 + \log_4 x + \log_4 16 = 6$, then value of x is

a 64 b 4 c 8 d 32

25. If $S_n = \frac{1}{6.11} + \frac{1}{11.16} + \frac{1}{16.21} + \dots$ to n terms then $6S_n$ equals

a $\frac{5n-4}{5n+6}$ b $\frac{n}{5n+6}$

b $\frac{2n-1}{5n+6}$ d $\frac{1}{5n+6}$

26. The remainder obtained when $1!^2 + 2!^2 + 3!^2 + \dots + 100!^2$ is divided by 10^2 is

a 27 b 28

c 17 d 14

27. In the group $G = \{1, 5, 7, 11\}$ under multiplication modulo 12, the solution of $7^{-1} \otimes_{12} x \otimes_{12} 11 = 5$ is equals

a 5 b 1

c 7 d 11

28. A subset of the additive group of real numbers which is not a subgroup is

a $\{0\}, +$ b $\mathbb{Z}, +$

(c) (\mathbb{N}, \cdot) d (d)

29. If $\vec{p} = \hat{i} + \hat{j}$, $\vec{q} = 4\hat{k} - \hat{j}$ and $\vec{r} = \hat{i} + \hat{k}$, then the unit vector in the direction of $3\vec{p} + \vec{q} - 2\vec{r}$ is

a $\frac{1}{3} \hat{i} + 2\hat{j} + 2\hat{k}$

b $\frac{1}{3} (\hat{i} - 2\hat{j} - 2\hat{k})$

c $\frac{1}{3} (\hat{i} - 2\hat{j} + 2\hat{k})$

d $\hat{i} + 2\hat{j} + 2\hat{k}$

30. If \vec{a} and \vec{b} are the two vectors such that $|\vec{a}| = 3$, $|\vec{b}| = 4$ and $|\vec{a} + \vec{b}| = 7$, then the angle between \vec{a} and \vec{b} is

a 120° b 60°

c 30° d 150°

31. if \vec{a} is vector perpendicular to both \vec{b} and \vec{c} , then

a $\vec{a} + \vec{b} + \vec{c} = \vec{0}$

b $\vec{a} \times \vec{b} + \vec{c} = \vec{0}$

c $\vec{a} \times \vec{b} \times \vec{c} = \vec{0}$

d $\vec{a} \cdot \vec{a} \times \vec{c} = \vec{0}$

32. If the area of the parallelogram with \vec{a} and \vec{b} as two adjacent sides is 15 sq unit, then the area of the parallelogram having, $3\vec{a} + 2\vec{b}$ and $\vec{a} + 3\vec{b}$ as two adjacent sides in sq unit is

a 120 b 105

c 75 d 45

33. if the lines $x+3y-9 = 0$, $4x+by-2 = 0$ and $2x-y-4 = 0$ are concurrent, then b equals

a -5 b 5

c 1 d 0

34. The equation of the circle having $x-y-2 = 0$ and $x-y+2 = 0$ as two tangents and $x-y=0$ as a diameter is

a $x^2 + y^2 + 2x - 2y + 1 = 0$

b $x^2 + y^2 - 2x + 2y - 1 = 0$

c $x^2 + y^2 = 2$

d $x^2 + y^2 = 1$

35. A circular sector of perimeter 60 m with maximum area is to be constructed. The radius of the circular arc in meter must be

a 20 b 5

c 15 d 10

36. $\frac{x^3+3x^2+3x+1}{(x+1)^5} dx$ is equal to

a $-\frac{1}{(x+1)} + c$ b $\frac{1}{5} \log x + 1 + c$

c $\log x + 1 + c$ d $\tan^{-1} x + c$

37. $\frac{\csc x}{\cos^2 x + \log \tan \frac{x}{2}}$ dx is equal to

a $\sin^2 [1 + \log \tan \frac{x}{2}] + c$

b $\tan\left[1+\log \tan \frac{x}{2}\right] + c$

c $\sec^2\left[1+\log \tan \frac{x}{2}\right] + c$

d $-\tan\left[1+\log \tan \frac{x}{2}\right] + c$

38. The complex number $\frac{(-\sqrt{3+3i})(1-i)}{3+\sqrt{3i} + i \sqrt{3+\sqrt{3i}}}$ when represented in the argand diagram is

a in the second quadrant

b in the first quadrant

c on the y -axis imaginary axis

d on the x -axis real axis

39. If $2x = -1 + \sqrt{3}i$, then the value of $1 - x^2 + x^6 - 1 - x + x^2 - x^6$ is equal to

a 32 b -64

c 64 d 0

40. The modulus and amplitude of $1 + i \sqrt{3}$ are respectively

a 2 and $\frac{\pi}{3}$ b 256 and $\frac{2\pi}{3}$

c 2 and $\frac{2\pi}{3}$ d 256 and $\frac{8\pi}{3}$

41. The value of $\lim_{x \rightarrow 0} \frac{5^x - 5^{-x}}{2x}$ is

a log 5 b 0

c 1 d 2 log 5

42. Which one of the following is not true always ?

a if $f(x)$ is not continuous at $x=a$, then it is not differentiable at $x=a$

b If $f(x)$ is continuous at $x=a$, then it is differentiable at $x=a$

c If $f(x)$ and $g(x)$ are differentiable at $x=a$, then $f(x) + g(x)$ is also

differentiable at $x=a$

d If a function $f(x)$ is continuous at $x=a$, then $\lim_{x \rightarrow a} f(x)$ exists

43. $\frac{dx}{x\sqrt{x^6-16}}$ is equal to

a $\frac{1}{3}\sec^{-1}\left(\frac{x^3}{4}\right) + c$ b $\cosh^{-1}\left(\frac{x^3}{4}\right) + c$

c $\frac{1}{12}\sec^{-1}\left(\frac{x^3}{4}\right) + c$ d $\sec^{-1}\left(\frac{x^3}{4}\right) + c$

44. If $I_1 = \int_0^{\pi/2} x \sin x \, dx$ and $I_2 = \int_0^{\pi/2} x \cos x \, dx$, then which one of the following is true ?

a $I_1 + I_2 = \frac{\pi}{2}$ b $I_1 - I_2 = \frac{\pi}{2}$

d $I_1 + I_2 = 0$ d $I_1 = I_2$

45. If $f(x)$ is defined $[-2, 2]$ by $f(x) = 4x^2 - 3x + 1$ and $g(x) = \frac{f(-x) - f(x)}{x^2 + 3}$, then $\int_{-2}^2 g(x) \, dx$ is equal to

a 64 b -48 c 0 d 24

46. The area enclosed between the parabola $y = x^2 - x + 2$ and the line $y = x + 2$ in sq unit equals

a $\frac{8}{3}$ b $\frac{1}{3}$

c $\frac{2}{3}$ d $\frac{4}{3}$

47. The solution of the differential equation $e^{-x}y + 1 \, dy + \cos^2 x + \sin^2 x \, y \, dx = 0$ subjected to the condition that $y=1$ when $x=0$ is

a $Y + \log y + e^x \cos^2 x = c$

b $\log y + 1 + e^{-x} \cos^2 x = 1$

c $Y + \log y = e^x \cos^2 x$

d $y + 1 + e^{-x} \cos^2 x = 2$

48. If the curve $y = 2x^3 + ax^2 + bx + c$ passes through the origin and the tangents drawn to it at $x = -1$ and $x = 2$ are parallel to the x axis, then the values of a, b and c are respectively

a 12, -3 and 0 b -3, -12 and 0

c -3, 12 and 0 d 3, -12 and 0

49. The locus of the point which moves such that the ratio of its distance from two fixed point in the plane is always a constant $k (< 1)$ is

a hyperbola b ellipse

c straight line d circle

50. The circles $ax^2+ay^2+2g_1x+2f_1y+c_1=0$ and $bx^2+by^2+2g_2x+2f_2y+c_2=0$ $a \neq 0$ and $b \neq 0$ cut orthogonally if

a $g_1g_2 + f_1f_2 = ac_1 + bc_2$

b $2g_1g_2 + f_1f_2 = bc_1 + ac_2$

c $bg_1g_2 + af_1f_2 = bc_1 + ac_2$

d $g_1g_2 + f_1f_2 = c_1 + c_2$